There are two Parts, assigned 2 point each, for a total of 4 points.

Part I (2 points):
A feedback system is shown in Figure 1 where $P(s)$ is a system model and $e^{-\tau s}$ represents transmission delay in the feedback path. The system is open loop stable and its transfer func-

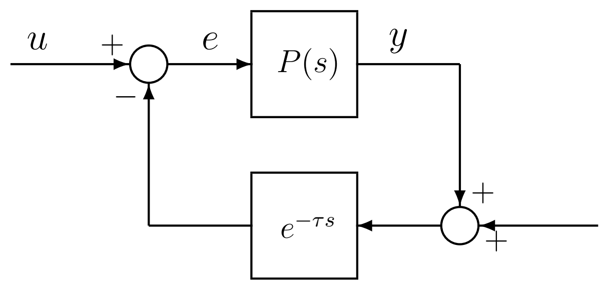

Figure 1: Feedback system.

tion can be fairly accurately modeled over the range of frequencies that are relevant to stability analysis by

$$G(s) = \sqrt{\frac{\sqrt{2}}{s+1}}$$

for $s = j\omega$ with ω measured in radians/sec. It is noted that the system model does have a rational transfer function, and hence it does not have a finite-dimensional state representation.

Complete/answer the following:
i) Draw the Bode plot for the open loop transfer function (approximately).
ii) Draw the Nyquist plot.
iii) Explain why the closed loop system is stable for zero time-delay $\tau = 0$.
iv) What is the maximal interval $[0, \tau_{\max})$ for the time-delay in the feedback loop for which the closed loop system remains stable.

Part II (2 points):
Consider two $n \times n$ real matrices A and Δ.

i) Consider the autonomous linear dynamical system

$$\dot{x}(t) \;=\; (A \,+\, \Delta)\, x(t),$$

with initial conditions $x(0) = x_0$. Show that provided $A\Delta = \Delta A$, the solution is given by

$$e^{\Delta t} e^{At} x_0.$$

ii) Give an example of two matrices A, Δ such that $A\Delta \neq \Delta A$.

iii) Let $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ and $\Delta = \begin{bmatrix} 0 & \delta_2 \\ \delta_1 & 0 \end{bmatrix}$.

- Determine the conditions that the positive scalars δ_1 and δ_2 have to satisfy to guarantee stability of the above system.
- Set $\delta_2 = 0$ and find the solution to the above system for the initial condition $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
- Set $\delta_2 = 0$ and consider the system with the input u and the output y

  $$\begin{aligned} \dot{x}(t) &= (A \,+\, \Delta)\, x(t) \,+\, B\, u(t) \\ y(t) &= C\, x(t) \end{aligned}$$

  where $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

  How does the peak value on the Bode magnitude plot of the transfer function (from u to y) depend on δ_1? At what frequency does this peak value take place?